Resonant Frequency of Uncovered and Covered Rectangular Microstrip Patch Using Modified Wolff Model

Anand K. Verma, *Member, IEEE,* and Zargham Rostamy

***Abstract*— A new model, called the Modified Wolff Model (MWM) is proposed to calculate the resonant frequency of a rectangular open microstrip patch, a patch with a dielectric superstrate and a patch inside a shielded enclosure. The model has also been applied to calculate resonant frequencies of higher order modes. Dependence of resonant frequency on the various parameters and its accuracy has been discussed using MWM. Results obtained from MWM follow very closely the results from various forms of fullwave analysis, and in almost all cases they are within 0.5% of published experimental results for the fundamental mode and within 1.7% for higher order modes. The method is computationally fast and efficient even on a desktop computer. The method could be used for other patches like circular, hexagonal and equilateral triangle.**

I. Introduction

THE RESONATING microstrip patch has been modelled as a cavity with magnetic walls along sides and electric walls on top and bottom. One of the primary advantage of the cavity model is the physical visualization of field distribution of various resonanting modes. The cavity model also provides an explicit expression for the resonant frequency. To get improved calculated frequency, the effective dielectric constant and extended length have been incorporated in the expression to account for the inhomogeneity of medium and fringe field, respectively. The empirical edge extension results of Hammerstad, Hammerstad and Jensen and Kirschning *et al.* have been attempted to improve the magnetic- wall cavity model [1]–[3]. In place of simple static formula for ε_{eff} Dearnly and Barel [4] have used an accurate dispersion model for $\varepsilon_{eff}(f)$ to get more accurate results for the resonant frequency. But the accuracy of cavity model remained around 3% to 5% for thin substrates and around 10% for thick substrates for the fundamental mode. Dearnley and Barel have also used the cavity model to calculate the resonant frequency of higher order modes with an accuracy around 3%. Wolff and Knoppik [5] replaced the ε_{eff} by dynamic dielectric constant to get improved results for the resonant frequency. Garg and Long [6] obtained a more accurate expression of line extension for the rectangular patch. This expression of extended length combined with $\varepsilon_{\rm dyn}$ provides more accurate result for the resonant frequency even for thick substrate 0.229 λ_d [6], [7].

Manuscript received November 26, 1991; revised April 23, 1992.
The authors are with the Department of Electronic Science, University of Delhi South Campus, Benito Juarez Road, New Delhi-21, India.
IEEE Log Number 9204023.

The fullwave analysis in its various forms has been used to calculate the resonant frequency of the rectangular patch. The method is applicable to open structure [8], [9], substrate-superstrate configuration [10] and shielded structure [11]. The fullwave analysis does not provide a physical picture of resonating modes and it requires a considerable amount of computation. Explicit formula for resonant frequency is absent, and resonant frequency is obtained from the solution of characteristic equations. Even though the method is rigorous, the calculated resonant frequency is not always very accurate.

We have reviewed the strengths and limitations of the cavity model and fullwave analysis. The cavity model is useful for designers. However, in its original form the cavity model is not applicable to the patches with superstrate and shielded patch. The authors have recently developed a MWM to account for the presence of superstrate [12], [13]. This paper discusses the physical basis of the modified Wolff model. The present model studies the effect of aspect ratio, substrate thickness, conductor thickness and uncertainties in the permittivity of substrate on the resonant frequency of the rectangular patch. The calculated resonant frequencies have been compared against the experimental results and the results obtained from various forms of fullwave analysis. The MWM has also been used to determine resonant frequency of higher order modes and the calculated resonant frequencies have been compared against the experimental results. The MWM model has further been extended to the patches with superstrate of low and high permittivity materials, and finally the model has been applied with success to the shielded resonator.

II. The Modified Wolff Model

The MWM is basically a cavity model. The original Wolff model was developed for a single layer open resonating structure [5]. To generalize the Wolff model to multilayer resonating structure shown in Fig. 1(a) we have adopted the variational method to calculate ε_{dyn}. Determination of ε_{dyn} takes into account the charge distribution along both the longitudinal and transverse directions of the patch. Thus, even though the MWM uses static variational method, it simulates the effect of fullwave analysis. Hence, the results obtained from MWM are comparable to the results obtained from that of the spectral domain analysis. The concept of dynamic dielectric constant also takes into account the fringe field along the radiating and non-radiating edges of the patch. The fringe

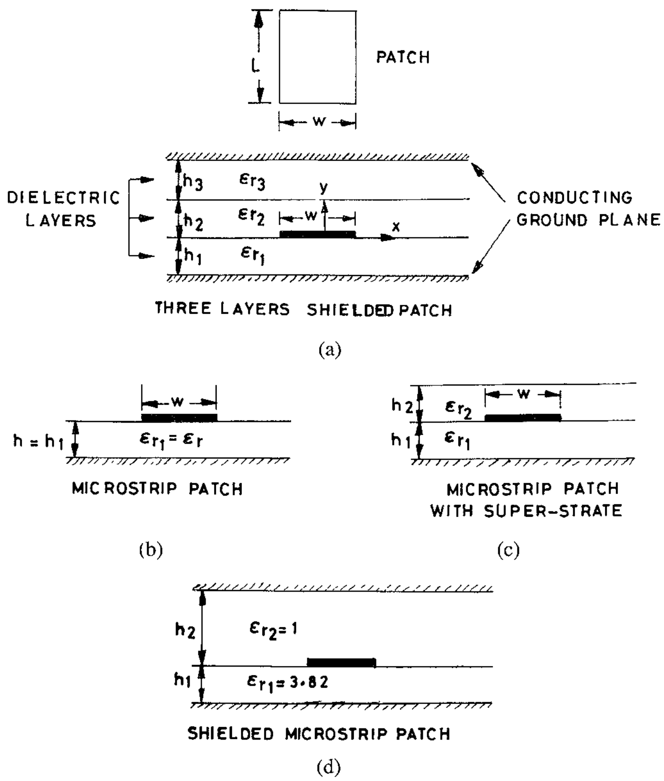


Fig. 1. (a) Three layers shielded patch. (b) Microstrip patch. (c) Microstrip patch with superstrate. (d) Shielded microstrip.

capacitance is influenced by the dielectric layers and modal field variation. Computation of the fringe capacitance for the multilayered structure assumes that the central capacitance of the patch is not affected by the superstrate. Effective length and effective width of the equivalent patch have been obtained from Garg and Long [6]. Effect of multilayer dielectrics on the effective length and width has also been accounted for by the use of variational method.

For the structure shown in the Fig. 1(a) the resonant frequency for mnth mode could be obtained from [2]

$$fr = fmn = \frac{Vo}{2\sqrt{\varepsilon_{dyn}}}\left[\left(\frac{n}{L_{eff}}\right)^2 + \left(\frac{m}{W_{eff}}\right)^2\right]^{1/2} \quad (1)$$

The dynamic permittivity of the covered patch is defined by [12], [13]:

$$\varepsilon_{dyn} = \frac{C_{dyn}(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3})}{C_{dyn}(\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1)} \quad (2)$$

where

$$C_{dyn}(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}) \quad \text{and} \quad C_{dyn}(\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1)$$

are the total dynamic capacitances of the patch in the presence of dielectric layers and the dielectric layers replaced by air, respectively. The total dynamic capacitance can be obtained from

$$\begin{aligned} &C_{dyn}(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}) \\ &= Co, dyn(\varepsilon_{r1}) + 2C_{e1}, dyn(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}) \\ &+ 2C_{e2}, dyn(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}) \end{aligned} \quad (3)$$

where $Co, dyn(\varepsilon_{r1})$ is the central dynamic capacitance of the patch due to the field just below the patch, which is not influenced by the dielectric cover or conducting shield. The central capacitance can be obtained from

$$Co, dyn(\varepsilon_{r1}) = \frac{Co, stat(\varepsilon_{r1})}{\gamma n \gamma m} \quad (4)$$

where,

$$Co, stat(\varepsilon_{r1}) = \frac{\varepsilon_{r0}\varepsilon_{r1} LW}{h_1} \quad (5)$$

$$i = n, m$$

$$\gamma_i = \begin{cases} 1 & \text{for } i = 0 \\ 2 & \text{for } i \neq 0 \end{cases}$$

The dynamic fringe capacitances, taking into account the modal field variation along the length L and the width W of patch are given by

$$C_{e1}, dyn(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}) = \frac{C_{e1}, stat(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3})}{\gamma_n}$$

and

$$C_{e2}, dyn(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}) = \frac{C_{e2}, stat(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3})}{\gamma_m}, \quad (6)$$

respectively. The static fringe capacitance could be obtained by subtracting central patch capacitance from the total capacitance obtained from the variational expression as follows:

$$C_{e1}, stat(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}) = \frac{1}{2}[CL - Co, stat(\varepsilon_{r1})] \quad (7)$$

$$C = \frac{Z}{VoZo^2} \quad (8)$$

where

- Z characteristic impedance of line with multilayer dielectrics and width W of conducting strip
- Z_o characteristic impedance of line with air dielectric and width W of conducting strip
- V_o velocity of light
- C capacitance per unit length of patch including fringe capacitance along two sides.

Likewise, $C_{e2}, stat(\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3})$ could be calculated by exchanging L for W. The total capacitance C per unit length of the line shown in Fig. 1(a) can be obtained by the use of combined variational method and TTL method in the Fourier domain [14], [17]

$$\frac{1}{C} = \frac{1}{\pi\varepsilon_0}\int_0^\infty \frac{[f(\beta)/Q]^2}{\beta h_1}\frac{1}{Y}d(\beta h_1) \quad (9)$$

where

$$Y = \varepsilon_{r1}\coth(\beta h_1) + \varepsilon_{r2}\left[\frac{\varepsilon_{r3}\coth(\beta h_3) + \varepsilon_{r3}\tanh(\beta h_3)}{\varepsilon_{r2} + \varepsilon_{r3}\tanh(\beta h_2)\coth(\beta h_3)}\right] \quad (10)$$

The admittance functions for open microstrip patch, Fig. 1(b), dielectric covered patch, Fig. 1(c), and shielded patch, Fig. 1(d) could be obtained from (10).

Q total charge on the strip
$f(\beta)$ Fourier transform of the charge distribution on the conducting strip assumed by Yamashita [15]
β Fourier variable
Y the admittance function of the line structure obtained by TTL method [14].

Following Garg and Long [6] the effective length L_{eff} and effective width W_{eff} can be obtained from

$$L_{eff} = L + \left[\frac{(W_{eq} - W)}{2}\right] \frac{\varepsilon_{eff}(W) + 0.3}{\varepsilon_{eff}(W) - 0.258} \tag{11}$$

where W_{eq} is the equivalent width determined from the planar waveguide model

$$W_{eq} = \frac{120\pi h_1}{Z\varepsilon_{eff}(W)} \tag{12}$$

Z is the characteristic impedance of the line with multilayer dielectrics and $\varepsilon_{eff}(W)$ is the effective dielectric constant of the same structure from the width side. $\varepsilon_{eff}(W)$ could be obtained from the variational method discussed above and is given by

$$\varepsilon_{eff}(W) = \left[\frac{Zo(W, h_1, h_2, h_3, \varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_{r3} = 1)}{Z(W, h_1, h_2, h_3, \varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3})}\right]^2. \tag{13}$$

Likewise, W_{eff} could be calculated by exchanging L for W and calculating L_{eq} and $\varepsilon_{eff}(L)$.

The effect of conductor thickness on the resonant frequency can also be incorporated in the model. Bahl and Garg [16] have reported the closed form expressions for dependence of characteristic impedance and effective dielectric constant of microstrip line on the finite thickness of strip conductor. They have claimed the accuracy within 2% for Z and ε_{eff} in the range $o \leq t/h_1 \leq 0.2, 0 \leq w/h_1 \leq 0.2$ and $\varepsilon_{r1} \leq 16$. In all calculations discussed above, we can replace physical L and W by the increased length Le, and the increased width We to take into account the effect of finite conductor thickness:

$$\frac{We}{h_1} = \frac{W}{h_1} + \frac{1.25}{\pi}\frac{t}{h_1}[1 + Ln(2h_1/t)], \frac{W}{h_1} \geq \frac{1}{2\pi}. \tag{14}$$

The effective dielectric constant calculated from (13) is reduced due to the conductor thickness and is given by

$$\varepsilon'_{eff} = \varepsilon_{eff} - \left[\frac{\varepsilon'_r - 1}{4.6}\right] \frac{t/h_1}{\sqrt{W/h_1}} \tag{15}$$

where, ε'_r is the equivalent permittivity of substrate of thickness h_1 of the equivalent single layer structure into which a multilayer structure could be reduced [17]. The increase in We causes decrease in Z and also decrease in the resonant frequency. Whereas, the decreased value of ε_{eff} causes increase in the resonant frequency. Thus, the overall effect of the conductor thickness on fr is small.

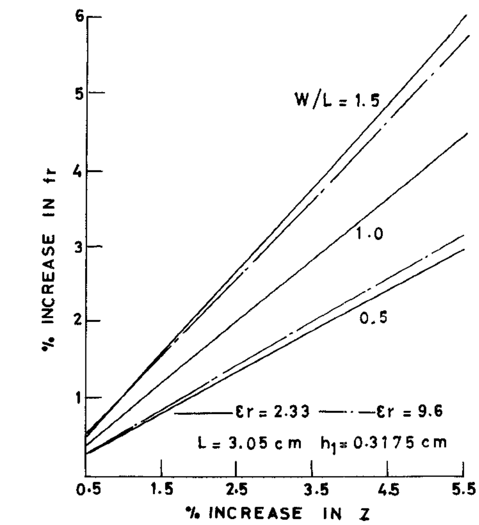


Fig. 2. X-axis: % increase in Z. Y-axis: % increase in fr. Variation of fr due to variational error in characteristic impedance.

III. Impedance Correction in MWM

Calculation of $\varepsilon_{dyn}, L_{eff}$ and W_{eff} involves determination of the characteristic impedance of microstrip line under multilayer condition. This has been achieved by the variational method, which provides the upper bound to the true value [15]. Deviation of the variational characteristic impedance Z_{var} from the true value Z_{con} is small but it is a function of W/h_1. The calculated value of characteristic impedance by the variational method is not always above the result obtained from the conformal mapping method. Fig. 2 shows the percentage increase in the resonant frequency of the patch for various aspect ratio with the percentage increase in the characteristic impedance which is due to its variational formulation. Change in the resonant frequency of the patch due to error in the characteristic impedance is a function of both the aspect ratio and permittivity of the substrate. For $W/L > 1$, lower permittivity ($\varepsilon_r = 2.33$) shows more changes in fr, compared to that of the patch on the high permittivity substrate ($\varepsilon_r = 9.60$). But the change gets reversed for $W/L < 1$ and the changes are identical for $W/L = 1$. For $W/L = 1$, 2.5% error in the estimated value of the characteristic impedance results into 2% error in the calculated value of resonant frequency. Thus, impedance calculated by variational method should be corrected against the true value of the characteristic impedance to get more accurate calculated fr. For the present work we have accepted Wheeler's formula for the characteristic impedance given below as [7]:

$$Z(W, h_1, \varepsilon_{r1}) = \frac{377}{\sqrt{\varepsilon_{eff}(W)}}\left[\frac{W}{h_1} + 1.393 + 0.667\ln\left(\frac{W}{h_1} + 1.444\right)\right]^{-1} \tag{16}$$

for

$$\frac{W}{h_1} \geq 1,$$

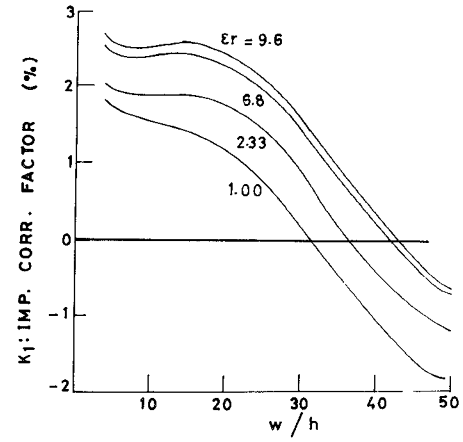


Fig. 3. X-axis: W/H. Y-axis: K_1: impedance corr. factor (%). Dependence of impedance correction factor: K_1 on W/H ratio.

and

$$\varepsilon_{eff}(W) = \frac{\varepsilon_{r1}+1}{2} + \frac{\varepsilon_{r1}-1}{2}\left[1+\frac{10}{W/h_1}\right]^{-(1/2)}. \quad (17)$$

The characteristic impedance of microstrip line calculated by the variational method and compared against Wheeler's formula generates a correction factor K_1. Thus, for further calculation characteristic impedance calculated by the variational method is reduced by this percentage correction factor which is given by

$$K_1 = \left[\frac{Z_{con} - Z_{var}}{Z_{con}}\right] \times 100 \quad (18)$$

Such correction factor for the uncovered patch made on the substrate $\varepsilon_r = 1, 2.33, 6.8, 9.6$ for different W/h_1 is shown in Fig. 3.

For larger W/L ratio, K_1 becomes negative, i.e., Z_{var} is less than Z_{con}. It happens due to the approximate nature of (16) and convergency problem of the improper integral in the variational method. To improve the calculation more accurate impedance reference [18] is needed and integral has to be evaluated more carefully. As a practical measure if K_1 becomes negative for a particular microstrip patch we can ignore the correction factor. This approximation works satisfactorily except for $\varepsilon_{r1} = 1$ and for the extremely wide patch $(W/L > 2)$.

The impedance correction factor K_1 for the covered and shielded microstrip patches also depends upon the permittivity of superstrate and its thickness and the height of shield. For the substrate-superstrate configuration, dependence of the correction factor K_1 on the thickness and permittivity of superstrate is shown in Fig. 9. Finally, the effect of change in K_1 on the resonant frequency of the patch due to the superstrate is shown Fig. 10. Thus, as a working approximation we may use the same K_1 which is obtained for the open microstrip patch.

IV. Open Microstrip Patch

The resonant frequency of the open microstrip patch, Fig. 1(b) has been calculated for the substrate of different thicknesses, the patch with various aspect ratios and the patch on the substrate of different permittivities. The calculated results by MWM have been compared against the published experimental results and the results obtained from the fullwave analysis.

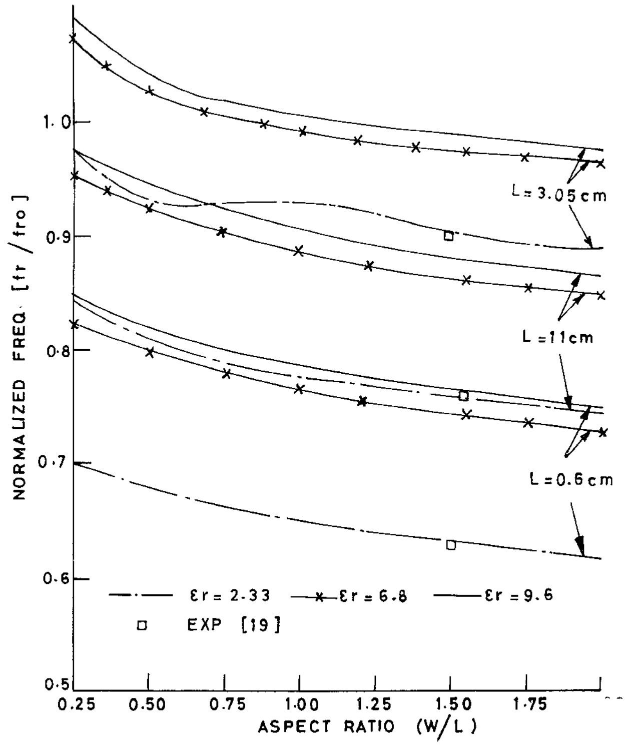


Fig. 4. X-axis: aspect ratio (W/L). Y-axis: normalized freq. $[fr/fro]$. Effect of aspect ratio on fr.

Aspect Ratio

Fig. 4 shows behavior of the normalized resonant frequency with respect to aspect ratio (W/L) in the range $1 < W/L < 2$. Normalization has been done by the zeroth order resonant frequency $fro = 15/L\sqrt{\varepsilon_{r1}}$ GHz, L in cm. The patches on $\varepsilon_{r1} = 2.33$ for $W/L = 1.5$ show agreement with experimental results within 0.7% [19]. To improve the directive gain of the patch radiator, Bhattacharya has recently analyzed a patch for $W/L = 7.34$ on $\varepsilon_{r1} = 2.32$ by using the generalized transmission line model [20]. His computed resonance frequency is 2.48 GHz against the measured value of 2.42 GHz, giving 2.49% error, whereas for the same patch MWM gives resonant frequency 2.39 GHz i.e., 1.24% error. In this calculation the impedance correction factor has not been used as for such a wide patch the reference impedance is not very accurate. Likewise, Newman and Tulyathan have calculated resonant frequencies for the patches with $L = 4.14$ cm, $h_1 = 0.1524$ cm, $\varepsilon_{r1} = 2.5$ and $1 < W/L < 2.61$ by using the method of moments [21]. Their results are within 1.89% compared against the measured values. Whereas, MWM provides the results within 0.4%. Thus, for the calculation of resonant frequency of rectangular patch the method of moment may not be a more accurate method.

Substrate Thickness

Fig. 5 shows decrease in the normalized resonant frequency fr/fro of patch with increase in electrical thickness of the

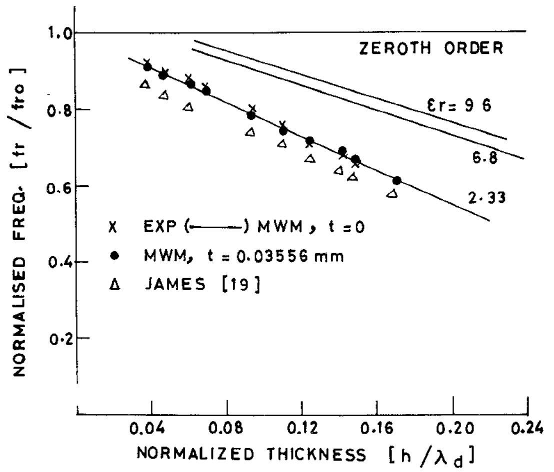


Fig. 5. X-axis: normalized thickness $[h/\lambda_d]$. Y-axis: normalized freq. $[fr/fro]$. Effect of conductor and substrate thicknesses on fr.

substrate. Thickness of substrate has been normalized by $\lambda_d = 30/fro\sqrt{\varepsilon_{r1}}$, where fro is in GHz, λ_d is in cm. The decrease in resonant frequency is more for the patch on low dielectric substrate. For the substrate thickness (h_1/λ_d) between 0.037 and 0.229, the experimental results of Chang *et al.* [19] for the patch on 3M Cuclađed 233 support the calculated results of MWM. Using James formula the calculated results of Chang *et al¿.* are around 4% higher than the measured values, whereas using Hammerstad's formula, the calculated results are around 8% lower than the measured values. Fig. 6(a) shows the deviation K_2 of the calculated resonant frequency by Dearnley and Barel [4], Chew and Liu [9] and MWM from the experimental results of Chang *et al¿.* Dearnley and Barel have used more accurate result of Krischning and Jansen on the extended length and frequency dependent $\varepsilon_{eff}(f)$. However, they got the accuracy only around 5%. This clearly shows that the concept of ε_{eff}, even by taking the dispersion into account is not compatible with the experimental results. Chew and Liu followed the integral equation formulation with Galerkin's method. Their results have rms, average, max., and min. errors 0.9%, 2.24%, 4.75%, and 0.57%, respectively. For the same structure the MWM calculates rms, average, max., and min. errors as 0.59%, 0.47%, 2.7% and 0.2% respectively. However, the nature of deviation K_2 obtained by the Chew and Liu and MWM is identical. This clearly indicates that the MWM, even though uses the static method simulates the fullwave analyses as it takes into account the charge distribution along both the longitudinal and transverse directions.

It would be illustrative to compare the calculated results of MWM with that of the results obtained by Itoh and Menzel using the spectral domain analysis [8] for the patch ($L = 1$ cm, $W = 1.5, h_1 = 0.158$ cm) on RT Duriod ($\varepsilon_{r1} = 2.35$). Their calculated value of resonant frequency is 8.29 GHz which, compared against the measured resonant frequency 8.41 GHz for the weak coupling provides 1.4% error. For the same patch MWM calculates resonant frequency 8.35 GHz, i.e., only 0.72% error.

Long *et al.* [19] also carried out their investigation on the effect of electrical thickness of the substrate on the resonant frequency for the patch made of aluminium sheet

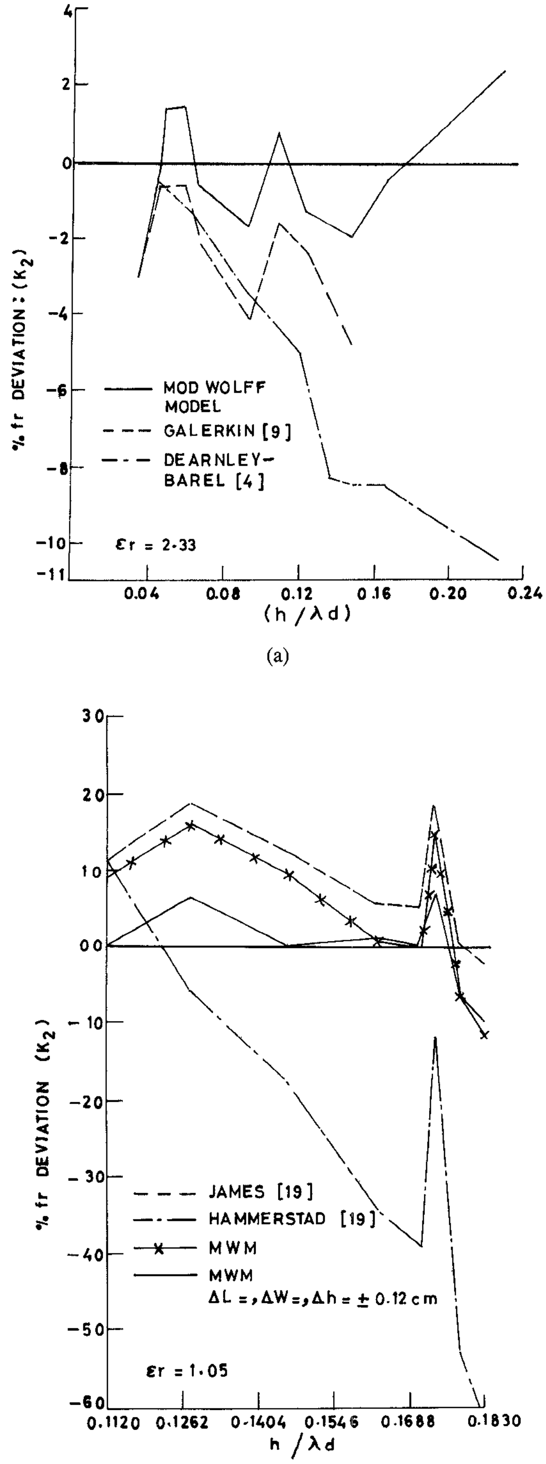
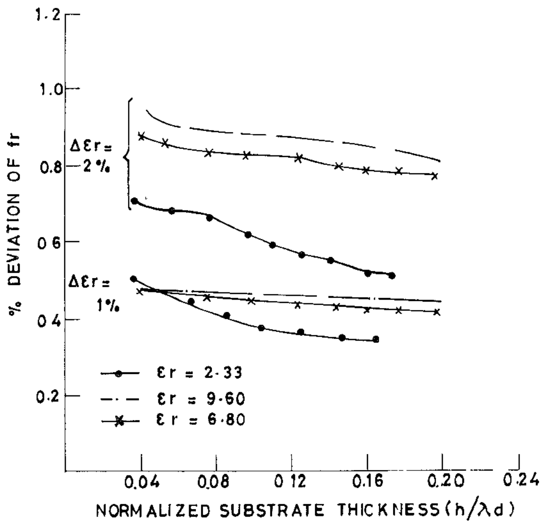


Fig. 6. (a) X-aixs: (h/λ_d). Y-axis: $\%fr$ deviation: (K_2). Comparison of method to calculate resonant freq. against experimental fr. (b) X-axis: h/λ_d. y-axis: $\%fr$ deviation (K_2). Comparison of methods to calculate resonant freq. against experimental fr. *(Fig. 6(c) continued on next page.)*

(1.78 cm × 2.67 cm, $t = 0.16$ cm) supported by styrofoam ($\varepsilon_{r1} = 1.05$) substrate. They have compared the measured resonant frequency against the results obtained from formulae of James and Hammerstad. None of the subsequent researchers, who have used the other table of Long *et al.* for $\varepsilon_{r1} = 2.33$ have presented their results on this patch. Fig. 6(b) shows



(c)

Fig. 6 (*Continued*). (c) X-axis: normalized substrate thickness (h/λ_d). Y-axis: % deviation of fr. Effect of variation in ε_r on resonant freq.

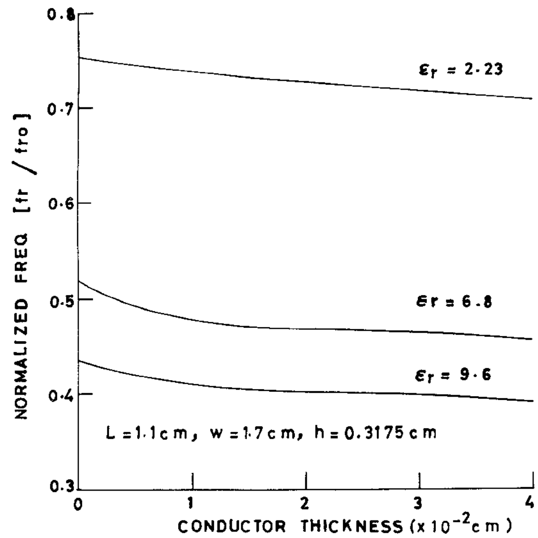


Fig. 7. X-axis: conductor thickness $(x10^{-2}$ cm). Y-axis: normalized freq. $[fr/fro]$. Effect of conductor thickness on fr.

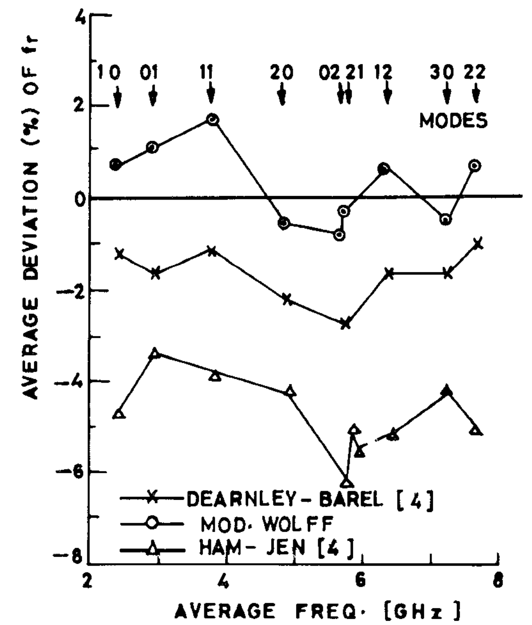


Fig. 8. X-axis: average freq. [GHz]. Y-axis: average deviation (%) of fr. Comparison of methods for resonant freq. of higher order modes against experimental fr.

deviation K_2 calculated by Hammerstad, James and MWM. Hammerstad provides an average error −26.3%, James 8.82% and MWM 3.96%. If the fabricational tolerances in L, W, and h_1 is taken as ±0.12 cm, then the average error in the calculated fr by MWM is only 0.11%. The behavior of the patch at $h_1/\lambda_d = 0.178$ is abrupt. It may be due to the uncertainty during experiment. Results of James and MWM show identical behavior.

Tolerance in Permittivity

Fig. 6(c) shows the effect of tolerance in ε_{r1} on the resonant frequency in the frequency range 1.5 GHz–11 GHz. For ±1% change in ε_{r1}, the variation of frequency is always within +0.5% for both the low and high permittivity substrates. Moreover, the variation in resonant frequency decreases with the increase in the electrical thickness of the substrate. The usual cavity model shows that the variation in resonant frequency is nearly independent of the electrical thickness of the substrate for frequency above 2.5 GHz and variation is more on the substrate of high permittivity [25]. This feature of MWM is useful for the accurate design of the patch at the higher frequency.

Conductor Thickness

Effect of thickness of conducting patch on the resonant frequency obtained from MWM is shown in Fig. 7. Initially, say for $t = 0.001$ cm decrease in the normalized frequency is more for the patch on the high dielectric substrate. However, up to 10 GHz such effect is not very significant as shown in Fig. 5 for the patches of Long *et al.* We have taken $t = 0.003556$ cm in our calculation. However, accounting the conductor thickness brings the calculated resonant frequency by MWM closer to the experimental result.

Higher Order Mode

Dearnley and Barel [4] have carried out measurement on the resonant frequency for the higher-order modes on nine rectangular patches made on 3M Cuclad 233 substrate. They have computed the resonant frequency for the higher order modes using Kirschning Jansen dispersion model for $\varepsilon_{eff}(f)$ and open-end effect. They have also computed the resonant frequencies of these modes using Hammerstad and James model. We have computed the resonant frequencies of these modes using MWM. All results are shown in Fig. 8. Results obtained from Kirschning–Jansen model is within 3% with average error 0.91% for all modes; Hammerstad–Jensen model gives higher average error i.e., 1.74% and the error goes above 6%. However, the deviation calculated using MWM is always within 1.71% with average error 0.28% for all modes. Therefore, modified Wolff model (MWM) is equally applicable to the calculation of resonant frequency of the higher order modes.

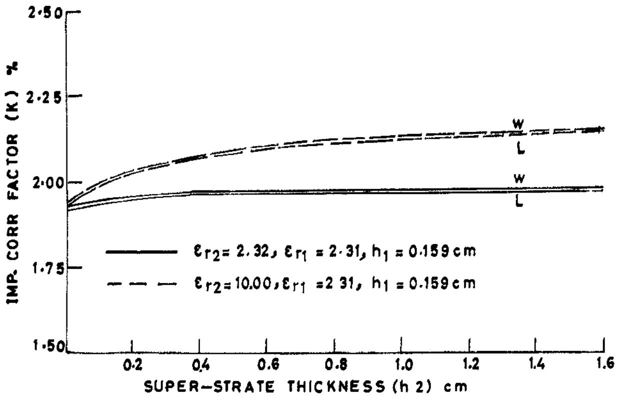


Fig. 9. X-axis: super-strate thickness (h_2) cm. Y-axis: imp. corr. factor $(K)\%$. Effect of superstrate on impedance correction factor.

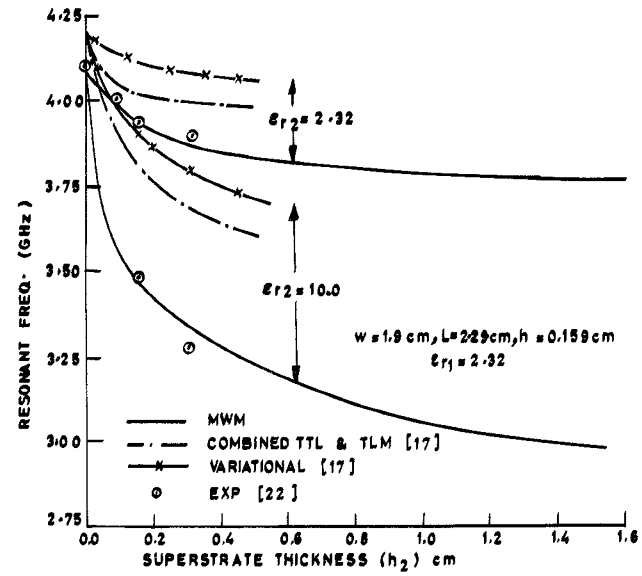


Fig. 10. X-axis: superstrate thickness (h_2). Y-axis: resonant freq. (GHz). Effect of superstrate on resonant freq.

V. Microstrip Patch with Superstrate

Dielectric superstrate is used to protect the patch antenna against rain, snow, etc. [22]. It is also used to achieve beam symmetry [24] and higher gain. Battacharya and Tralman [23] have reported an empirical relation on the change of resonant frequency with the thickness of superstrate for rectangular patch with an error between 1.34% to 7.8% compared to the experimental results [17]. Bahl *et al.* [22] attempted the problem by using the variational method for the rectangular patch on the RT-duriod with superstrate of duriod $(\varepsilon_{r2} = 2.32)$, plexiglass, $(\varepsilon_{r2} = 2.6)$, mylar $(\varepsilon_{r2} = 3.00)$, Epsilon $-10, (\varepsilon_{r2} = 10.2)$ and custom high $-K\,(\varepsilon_{r2} = 10)$. The variational method gives error in the calculated resonant frequency as high as 4.8% and 5.9% for the superstrate of RT-duriod and plexiglass, respectively and for the custom high $-K$ error is 16% [17]. Verma *et al.* [17] have reported a combined TTL–TLM method with improved results on the low dielectric superstrate but the error for custom high $-K$ still remained 13.16% [7]. Using the spectral domain analysis, Daniel *et al.* [10] calculated the resonant frequency of the

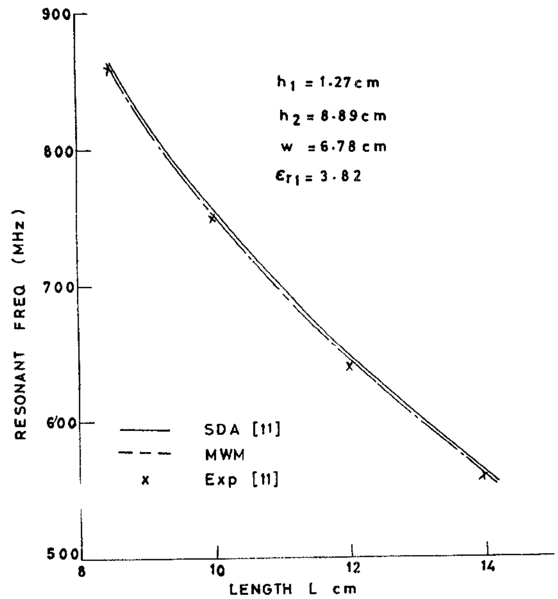


Fig. 11. X-axis: length L cm. Y-axis: resonant freq. (MHZ). Resonant freq. of shielded patch.

same patch with low permittivity superstrate. Their results are within 0.5% of the experimental values. However, they have not presented SDA result on the superstrate of custom high $-K$. The calculated resonant frequency using MWM alongwith results from other methods and experimental results are shown in Fig. 10. Even for the thick RT-duriod cover the MWM gives error within 0.5% whereas, error in the calculated fr by both the variational and combined TTL–TLM methods increases with the increase in thickness of the cover. The MWM result on custom high $-K$ shows remarkable improvement, as the error is always within 2.8%. The error comes down to 2.6% if the effect of the superstrate on the impedance correction factor is taken into account. This could be achieved by converting the double layer structure into an equivalent single layer structure [17]. Effect of superstrate on the impedance correction factor K_1 is shown in Fig. 9. For the thick and high dielectric constant materials somewhat more improvement in the result may follow if the effect of superstrate on the charge distribution function of Yamashita [15] could also be considered.

VI. Shielded Patch

The cavity model is not applicable to the shielded patch. However, the MWM also calculates the resonant frequency for such structure shown in Fig. 1(d). Dimension of the structure has been taken from Itoh [11]. Fig. 11 shows the calculated resonant frequency by MWM and SDA and the measured frequency. The results of MWM follow more closely the experimental results as compared to results from the SDA. Both methods calculate the resonant frequency within 0.5% of the experimental value. The impedance correction factor for this patch has been calculated as 1.8% for the patch with dielectric $(\varepsilon_{r1} = 3.82)$ and 1.2% for the patch with air dielectric. Final impedance correction factor K_1 has been obtained on converging the result for K_1 while increasing the height of the cover.

VII. Conclusion

The modified Wolff model (MWM) extends the concept of cavity model to the multilayer resonating structure and shielded patch. It achieves the accuracy of fullwave analysis without any computational difficulties. MWM results have been compared against large number of published experimental and fullwave analysis results. Almost in all cases, MWM results follow more closely the experimental results compared to that of the results of fullwave analysis. Accuracy of fullwave analysis either in the space domain or in the Fourier domain is constrained by the computational difficulties. The MWM has also been applied satisfactorily for the higher order modes, where fullwave results are not available. The MWM has been successfully applied to the covered circular patch [13]. We have also applied the MWM to the triangular and hexagonal patches. Results on these and other patches will be reported separately. Thus, MWM is a useful tool for the microwave CAD on patch antenna and resonators.

Acknowledgment

We are thankful to Prof. B. Bhat and Prof. S. K. Kaul for useful discussions. We are also thankful to Prof. P. C. Mathur, Prof. G. P. Srivastava, Dr. Laxman Prasad, Mr. P. S. Sundaram, Mr. K. R. P. Verma and Mr. B. Kumar for help in various ways and constant encouragement. We are also thankful to the reviewer for the suggestion to incorporate the effect of change in ε_r on fr.

References

[1] E. O. Hammerstad, "Equations for microstrip circuit design," *Proc. 5th European Microwave Conf.*, Sept. 1975, pp. 268–272.

[2] E. O. Hammerstad and O. Jensen, "Accurate models for computer aided design," in *1980 IEEE MTT-S Int. Microwave Symp. Dig.*, Washington, D.C., pp. 407–407.

[3] M. Kirschning, R. H. Jansen, and N. H. L. Koster, "Accurate model for open end effect of microstrip lines," *Electron. Lett.*, vol. 17, no. 3, pp. 123–125, 1981.

[4] R. W. Dearnley and A. R. F. Barel, "A comparison of models to determine the resonant frequencies of a rectangular microstrip antenna," *IEEE Trans. Antennas Propagat.*, vol. 37, no. 1, pp. 114–118, Jan. 1989.

[5] I. Wolff and N. Knoppik, "Rectangular and circular microstrip disk capacitors and resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, no. 10, pp. 857–864, 1974.

[6] R. Garg and S. A. Long, "Resonant frequency of electrically thick rectangular microstrip antenna," *Electron. Lett.*, vol. 23, no. 21, pp. 1149–1159, 1987.

[7] F. Abboud, J. P. Damiano, and A. Papiernik, "Simple model for the input impedance of Coax-fed rectangular microstrip patch antenna for CAD," *Proc. Inst. Elec. Eng.*, vol. 135, pt. H, no. 5, pp. 323–326, Oct. 1988.

[8] T. Itoh and W. Menzel, "A fullwave method analysis for open microstrip structures," *IEEE Trans. Antennas Propagat.*, vol. AP-29, no. 1, pp. 63–68, Jan. 1981.

[9] W. C. Chew and Q. Liu, "Resonance frequency of a rectangular microstrip patch," *IEEE Trans. Antennas Propagat.*, vol. 36, no. 8, pp. 1045–1056, Aug. 1988.

[10] J. P. Daniel, E. Penard, and C. Terret, "Design and technology of Low-Cost printed antenna," in *Handbook of Microstrip Antennas*, vol. 1, J. R. James and P. S. Hall, Eds. London: Peter Peregrinus Ltd., 1989, pp. 592–597.

[11] T. Itoh, "Analysis of microstrip resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, no. 11, pp. 946–952, Nov. 1974.

[12] A. K. Verma and Z. Rostamy, "Modified Wolff model for the resonant frequency of covered rectangular microstrip patch antenna," *Electron. Lett.*, vol. 27, no. 20, pp. 1850–1852, 1991.

[13] {—}{—}, "Modified Wolff model for the determination of resonant frequency of dielectric covered circular microstrip patch antenna," *Electron. Lett.*, vol. 27, no. 24, pp. 2234–2236, Nov. 21, 1991.

[14] R. Crampagne, M. Ahmadpanah and J.-L. Guiraud, "A simple method for determining the Green's function for a large class of MIC lines having multilayered dielectric structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 82–87, Feb. 1978.

[15] E. Yamashita, "Variational method for the analysis of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 529–535, Aug. 1968.

[16] I. J. Bahl and R. Garg, "Simple and accurate formula for a microstrip with finite strip thickness," *Proc. IEEE*, vol. 65, no. 11, pp. 1611, Nov. 1977.

[17] A. K. Verma, A. Bhupal, Z. Rostamy, and G. P. Srivastava, "Analysis of rectangular patch antenna with dielectric cover," *IEICE Trans.* (Japan), vol. E. 74, no. 5, pp. 1270–1276, May 1991.

[18] R. C. Callarotti and A. Gallo, "On the solution of a microstrip line with two dielectrics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-312, no. 4, pp. 333–338, Apr. 1984.

[19] E. Chang, S. A. Long and W. F. Richards, "An experimental investigation of electrically thick rectangular microstrip antenna," *IEEE Trans. Antennas Propagat.*, vol. AP-34, no. 6, pp. 767–772, June 1986.

[20] A. K. Bhattacharya, "Long rectangular patch antenna with a single feed," *IEEE Trans. Antennas Propagat.*, vol. 38, no. 7, pp. 987–993, July 1990.

[21] E. H. Newman and P. Tulyathan, "Analysis of microstrip antennas using moment methods," *IEEE Trans. Antennas Propagat.*, vol. AP-29, no. 1, pp. 47–53, Jan. 1981.

[22] I. J. Bahl, P. Bhartia, and S. S. Stuchly, "Design of microstrip antenna covered with a dielectric layer," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 314–318, Mar. 1982.

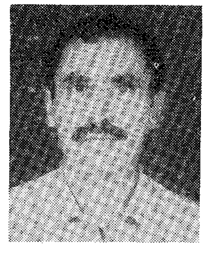
[23] A. K. Bhattacharya and T. Tralman, "Effect of dielectric superstrate on patch antenna," *Electron. Lett.*, vol. 24, no. 6, pp. 356–358, Mar. 1988.

[24] A. K. Verma, Z. Rostamy, S. Kumar, and G. P. Srivatava, "Symmetrical beamwidth suspended microstrip antenna," *Proc. Int. Conf. APMC-90*, Tokyo, Sept. 1990.

[25] I. J. Bahl and P. Bhartia *Microstrip antennas*. Dedham, MA: Artech House, 1980, pp. 66–69.

A. K. Verma (M'92) was born on Oct. 2, 1948. He received the B.Tech. in electronics and comm. engineering from B.I.T. Sindri, India in 1970 and Ph.D. in microwave eng. in 1984. He has been associated with the Radio TV transmitters, antenna and satellite communication. Presently he is a Reader with Dept. of Electronic Science and Delhi University, India. His research interest is in analysis and design of multilayered microstrip lines, resonators, and microstrip antennas.

Dr. Verma is Fellow of IETE, India.



Zargham Rostamy was born in Khallkhal, Iran in 1953. He received the B.Sc. degree in physics from the University of Calicut, India in 1986 and the M.Sc. in physics from the University of Meerut, India, in 1988. He is presently working towards the Ph.D. at Delhi University, and is involved with multilayered microstrip antennas and resonating structures.